

# FORMULAE USED TO COMPUTE TAC REDISTRIBUTIONS FOR HAKE AND SOLE TRAWL FISHERIES

## Symbol Glossary

- $T$  TAC for 2006 for the sector concerned (in tons)
- $p^i$  Applicant  $i$ 's proportional share of the 2005 TAC for the sector concerned
- $s_p^i$  Applicant  $i$ 's score for performance (jobs and investment)
- $s_T^i$  Applicant  $i$ 's score for transformation
- $s^i$  Applicant  $i$ 's total score
- $n$  Number of successful applicants
- $n^*$  Number of unsuccessful applicants
- $m = n + n^*$  Total number of applicants

## Deductions to Create Pool 1

### Constituents of formulae:

- $x$  The percentage of the 2006 TAC ( $T$ ) which is to be placed in Pool 1
- $s_c, v$  Parameters of the formulae
- $R^i$  Amount in tons by which the amount which applicant  $i$  would have received in the absence of any redistribution ( $p^i T$ ) is reduced to contribute to Pool 1.

$$\lambda^i = \begin{cases} 0 & \text{if } s^i \geq s_c \left(1 + \frac{1}{v}\right) \\ 1 - \frac{s^i - s_c}{s_c} v & \text{if } s^i \leq s_c \left(1 + \frac{1}{v}\right) \end{cases} \quad (1)$$

$$R^i = \lambda^i p^i \frac{x}{100} T / \left\{ \sum_{j=1}^m \lambda^j p^j \right\} \quad (2)$$

so that:

$$\sum_{i=1}^m R^i = \frac{x}{100} T \quad (3)$$

i.e. the sum of reductions from *all* applicants (not just the successful ones) to create Pool 1 is equal to  $x\%$  of the TAC as required.

Equations (1) and (2) are formulated such that the higher an applicant's total score ( $s$ ), the lesser the percentage of the allocation which the applicant would have received (in the absence of any redistribution) that is placed in Pool 1.

The specific parameter values chosen for the hake and sole trawl fisheries were:

$$\begin{aligned} x &= 10 \text{ (i.e. 10\% of the TAC to Pool 1)} \\ s_c &= 50 \\ v &= 2 \end{aligned} \tag{4}$$

so that the equations (1) and (2) become:

$$\lambda^i = \begin{cases} 0 & \text{if } s^i \geq 75 \\ 1 - \frac{s^i - 50}{25} & \text{if } s^i \leq 75 \end{cases} \tag{5}$$

$$R^i = \lambda^i p^i 0.1T / \left\{ \sum_{j=1}^m \lambda^j p^j \right\} \tag{6}$$

Note that formula (5) means that any applicant with a total score of 75 or more would make no contribution to Pool 1.

## Redistribution of Pool 1

Note that Pool 1 is to be redistributed amongst only those “*small*” applicants (“small” in terms of proportional holding of the 2005 TAC for the sector) who scored well on *both* performance and transformation.

Constituents of formulae:

$y$	Size exclusion criterion
$A, B$	Parameters of the formulae (both percentiles)
$p_p^i / p_T^i$	Applicant $i$ 's percentile on a distribution of performance/transformation scores ( $s_p / s_T$ ) over all <i>successful</i> applicants
$w_1^i$	A weight accorded to applicant $i$ in effecting allocation
$A_1^i$	Allocation (in tons) from Pool 1 to successful applicant $i$ .

For the purpose of this exercise, percentiles ( $p_{p/T}$ ) for scores ( $s_{p/T}$ ) were computed as follows:

- i) the scores were arranged in ascending order;
- ii) the lowest score was accorded a percentile  $100/n$ , the next lowest  $200/n$ , and so on so that the highest score was accorded a percentile of 100;
- iii) if two or more applicants achieved identical scores, all were accorded the same percentile, this being taken as the associated average (e.g. say entries 5 and 6 on the list of scores in ascending order had identical scores, both would be accorded a percentile of  $550/n$ ).

The weight for applicant  $i$  was then computed as:

$$w_1^i = \begin{cases} 0 & \text{if } p_p^i \leq A \text{ and } p_T^i \leq A \\ \frac{p_p^i - A}{B - A} \frac{p_T^i - A}{B - A} & \text{if } A \leq p_p^i \leq B \text{ and } A \leq p_T^i \leq B \\ \frac{p_p^i - A}{B - A} & \text{if } A \leq p_p^i \leq B \text{ and } p_T^i \geq B \\ \frac{p_T^i - A}{B - A} & \text{if } p_p^i \geq B \text{ and } A \leq p_T^i \leq B \\ 1 & \text{if } p_p^i \geq B \text{ and } p_T^i \geq B \end{cases} \quad (7)$$

except that:

$$w_1^i = 0 \quad \text{if } p^i \geq y \quad (8)$$

i.e. “big” applicants whose proportion of the 2005 TAC in the sector concerned exceeded  $y$  may not receive any allocation from Pool 1.

Then:

$$A_i^i = \left( \frac{x}{100} T \right) w_1^i / \sum_{j=1}^n w_1^j \quad (9)$$

The effect of equation (7) for successful applicants is as follows:

- i) those whose performance or transformation scores are less than the  $A$  percentile receive no allocation from Pool 1;
- ii) those whose performance and transformation scores are both greater than the  $B$  percentile receive a maximum allocation;
- iii) those scoring between these two extremes receive a proportion of the maximum such that the proportion is small if either score is close to the  $A$  percentile, but increases smoothly to 1 as both scores approach the  $B$  percentile.

The specific parameter value choices for the hake and sole trawl fisheries were:

$$\begin{aligned} y &= 0.15 && \text{(i.e. applicants with more than 15\% of the TAC for the sector in 2005 could not benefit from Pool 1)} \\ A &= 0.4 && \text{(i.e. 40\%)} \\ B &= 0.6 && \text{(i.e. 60\%)} \end{aligned} \quad (10)$$

## Creation of Pool 2

Pool 2 is made up of two contributions:

- i)  $D$  tons subtracted from each successful applicant irrespective of the size of their allocation;
- ii) Amounts which unsuccessful applicants would have received had they been successful, less any reduction therefrom for Pool 1.

Hence the total amount in tons in Pool 2 ( $T_2$ ) is given by:

$$T_2 = nD + \sum_{i=1}^{n^*} (p^i T - R^i) \quad (11)$$

The specific parameter value choices for the hake and sole trawl fisheries were:

Offshore hake	$D = 400$ tons	
Inshore hake	$D = 75$ tons	(12)
Sole	$D = 8$ tons .	

## Redistribution of Pool 2

Note that Pool 2 is to be redistributed amongst *all* successful applicants taking account of their total scores ( $s$ ), but independent of the size of their allocations.

Constituents of formulae:

$X, r, k$  Parameters of the formulae

$w_2^i$  A weight related to total score accorded to applicant  $i$  in effecting reallocation

$A_2^i$  Allocation (in tons) from Pool 2 to successful applicant  $i$ .

The weight  $w_2^i$  was calculated as follows:

$$w_2^i = \begin{cases} 0 & \text{for } 0 \leq s^i \leq X \\ 50 \left[ 1 - \left( \frac{1 - s^i/50}{1 - X/50} \right)^r \right]^k & \text{for } X \leq s^i \leq 50 \\ 50 \left[ 1 + \left( \frac{s^i/50 - 1}{\frac{100 - X}{50} - 1} \right)^r \right]^k & \text{for } 50 \leq s^i \leq (100 - X) \\ 100 & \text{for } (100 - X) \leq s^i \leq 100 \end{cases} \quad (13)$$

Then:

$$A_2^i = T_2 w_2^i / \sum_{j=1}^n w_2^j \quad (14)$$

The purpose of equations (13) is to accentuate the difference in allocations to higher and lower scoring applicants, by making them greater than would be the case were allocations made directly proportional to total score ( $s$ ). This is effected by some combination of  $X \geq 0$ ,  $r \leq 1$  and  $k \geq 1$ . Note that if  $X = 0$ ,  $r = 1$  and  $k = 1$ , equations (13) simplify to direct proportionality:

$$w_2^i = s^i \quad \text{for } 0 \leq s^i \leq 100 \quad (15)$$

The specific parameter value choices for the hake and sole trawl fisheries were:

$$\begin{aligned} X &= 0 \\ r &= 0.5 \\ k &= 1 \end{aligned} \quad (16)$$

i.e.:

$$w_2^i = \begin{cases} 50 \left[ 1 - \sqrt{1 - s^i/50} \right] & \text{for } 0 \leq s^i \leq 50 \\ 50 \left[ 1 + \sqrt{s^i/50 - 1} \right] & \text{for } 50 \leq s^i \leq 100 \end{cases} \quad (17)$$